Relation on gaugino masses in a supersymmetric $SO(10)_{GUT} \times SO(6)_H$ unified model

Kiichi Kurosawa
Department of Physics, University of Tokyo, Tokyo 113-0033, Japan
and Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8201, Japan

Yasunori Nomura and Koshiro Suzuki
Department of Physics, University of Tokyo, Tokyo 113-0033, Japan
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The doublet-triplet splitting problem in supersymmetric grand unified theories is elegantly solved in a supersymmetric $SO(10)_{GUT} \times SO(6)_H$ model. In this model, the gauginos in the supersymmetric standard model do not respect the usual grand unified theory (GUT) gaugino mass relation. We point out that in spite of nonunified gaugino masses, there is one nontrivial relation among gaugino masses in the model. Thus, it can be used to test the model in future experiments.
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A supersymmetric (SUSY) grand unified theory (GUT) [1] provides an elegant explanation for the stability of the weak scale against large radiative corrections and peculiar hypercharge assignments in the standard model (SM). It is supported by the fact that the observed three gauge coupling constants unify at a very high energy scale, $M_G \simeq 2 \times 10^{16}$ GeV [2], and various quark and lepton multiplets in the SM fit well into fewer multiplets of the GUT group such as $SU(5)_{GUT}$ or $SO(10)_{GUT}$.

SUSY GUT models, however, generically suffer from the ''doublet-triplet splitting problem.'' In a SUSY GUT, the Higgs doublets in the SM have their color-triplet partners. The masses for these Higgs triplets should be of the order of the GUT scale in order to ensure the stability of proton and/or successful gauge coupling unification, while those for Higgs doublets are of the order of the weak scale. This requires severe fine tuning between parameters in the minimal SUSY GUT models [1]. Among several mechanisms [3–8] proposed to solve the problem, one interesting possibility is to enlarge the gauge group to the semisimple one, $G_{GUT} \times G_H$ [5,6], where the doublet-triplet splitting problem is solved by the missing partner mechanism [3] without introducing large representations under the GUT group.

As shown in Ref. [9], this class of models may not respect the GUT gaugino mass relation

$$\frac{m_1}{\alpha_1} = \frac{m_2}{\alpha_2} = \frac{m_3}{\alpha_3}, \tag{1}$$

which is often considered a robust prediction of SUSY GUT. Here, α_1, α_2, and α_3 (m_1, m_2, and m_3) represent the gauge coupling constants (gaugino masses) for $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$, respectively.[1] In this Brief Report, we point out that in a supersymmetric unified model based on a semisimple gauge group $SO(10)_{GUT} \times SO(6)_H$ [6] there is a certain relation among gaugino masses in spite of nonunified gaugino masses, and thus it can be used to test the model by future experiments.

Let us first review the $SO(10)_{GUT} \times SO(6)_H$ model proposed in Ref. [6] briefly. We introduce eleven flavors of hyperquarks Q^A_α ($A = 1, \ldots, 11$; $\alpha = 1, \ldots, 6$) which transform as vector **6** representations under the hypercolor gauge group $SO(6)_H$. The first ten hyperquarks Q^I_α ($I = 1, \ldots, 10$) form vector **10** representations and the last one Q^{11}_α is a singlet of the $SO(10)_{GUT}$. We also introduce $SO(6)_H$-singlet chiral superfields, $H_I(\mathbf{10})$, $S_{IJ}(\mathbf{54})$, $A_{IJ}(\mathbf{45})$, $\phi(\mathbf{16})$, $\overline{\phi}(\mathbf{16}^*)$ and $\chi(\mathbf{1})$ ($I, J = 1, \ldots, 10$), where the numbers in parentheses denote transformation properties under the $SO(10)_{GUT}$. All the quark and lepton superfields constitute spinor **16** representations of the $SO(10)_{GUT}$ and singlets of the $SO(6)_H$.

In order to forbid unwanted tree-level mass terms such as $H_I H_I$ and $Q^{11}_\alpha Q^{11}_\alpha$ in the superpotential, we impose an anomalous global $U(1)_A$ symmetry

$$Q^I_\alpha \to Q^I_\alpha, \quad Q^{11}_\alpha \to e^{-2i\xi} Q^{11}_\alpha, \quad H_I \to e^{2i\xi} H_I,$$
$$S_{IJ} \to S_{IJ}, \quad A_{IJ} \to A_{IJ}, \quad \phi \to e^{-i\xi}\phi, \quad \overline{\phi} \to e^{i\xi}\overline{\phi}, \quad \chi \to \chi. \tag{2}$$

The tree-level superpotential is given by

$$W = \lambda_Q\, Q^I_\alpha Q^J_\alpha S_{IJ} + m_Q\, Q^I_\alpha Q^I_\alpha + h\, Q^I_\alpha Q^{11}_\alpha H_I + \tfrac{1}{2} m_S\, \mathrm{Tr}(S^2) + \tfrac{1}{3}\lambda_S\, \mathrm{Tr}(S^3) + m_A\, \mathrm{Tr}(A^2) + \lambda_A\, \mathrm{Tr}(A^2 S) + g_\phi (\overline{\phi}\sigma_{IJ}\phi) A_{IJ} + g_\chi (\overline{\phi}\phi - \mu^2)\chi. \tag{3}$$

Classically, there is an undesired vacuum $\langle S_{IJ} \rangle = \langle Q^I_\alpha \rangle = 0$, in which the gauge group is not broken down to the SM one. However, it does not exist quantum mechanically, since if $\langle S_{IJ} \rangle = 0$ the low-energy physics below the scale $m_Q \neq 0$ would be effectively described by an $SO(6)_H$ gauge theory with one massless hyperquark Q^{11}_α, and there is no stable SUSY vacuum in this case ($N_f \leq N_C - 5$) [10].

[1]Here, we take the GUT normalization, $\alpha_1 = \frac{5}{3}\alpha_Y$.

Therefore, S_{IJ} must have a vacuum expectation value (VEV) and indeed we can find the following desired SUSY vacuum which is stable quantum mechanically.

$$\langle S_{IJ}\rangle = v\begin{pmatrix} \frac{3}{2}\mathbf{1} &&&& \\ & \frac{3}{2}\mathbf{1} &&& \\ && -\mathbf{1} && \\ &&& -\mathbf{1} & \\ &&&& -\mathbf{1}\end{pmatrix},$$

$$\langle Q^I_\alpha\rangle = v_Q\begin{pmatrix} & \mathbf{0} & \\ \mathbf{1} && \\ & \mathbf{1} & \\ && \mathbf{1}\end{pmatrix}; \quad \mathbf{1}=\begin{pmatrix}1&0\\0&1\end{pmatrix},$$

$$\langle Q^{11}_\alpha\rangle = \langle H_I\rangle = 0, \tag{4}$$

$$\langle A_{IJ}\rangle = \begin{pmatrix} ai\sigma_2 &&&& \\ & ai\sigma_2 &&& \\ && bi\sigma_2 && \\ &&& bi\sigma_2 & \\ &&&& bi\sigma_2\end{pmatrix};$$

$$i\sigma_2 = \begin{pmatrix}0&1\\-1&0\end{pmatrix},$$

$$\langle\phi\rangle = v_\phi(\uparrow\otimes\uparrow\otimes\uparrow\otimes\uparrow\otimes\uparrow), \quad \langle\bar{\phi}\rangle = v_\phi(\downarrow\otimes\downarrow\otimes\downarrow\otimes\downarrow\otimes\downarrow).$$

In this vacuum the $SO(10)_{GUT}\times SO(6)_H$ is broken down to the Pati-Salam gauge group $SO(6)_C\times SU(2)_L\times SU(2)_R$ ($SO(6)_C\simeq SU(4)_{PS}$) [11] by the VEVs of S_{IJ} and Q^I_α, and it is further broken down to the $SU(3)_C\times SU(2)_L\times U(1)_Y$ by $\langle A_{IJ}\rangle$, $\langle\phi\rangle$, and $\langle\bar{\phi}\rangle$. From the F-flatness conditions of Eq. (3),

$$\begin{aligned} &-5m_S v + 2\lambda_A(a^2-b^2) - \tfrac{5}{2}\lambda_S v^2 + 2\lambda_Q v_Q^2 = 0,\\ &(\lambda_Q v - m_Q)v_Q = 0,\\ &(2m_A+3\lambda_A v)a - g_\phi v_\phi^2 = 0,\\ &(2m_A-2\lambda_A v)b - g_\phi v_\phi^2 = 0,\\ &g_\phi(4a+6b)v_\phi + g_\chi v_\phi\chi = 0,\\ &g_\chi(v_\phi^2-\mu^2) = 0,\end{aligned} \tag{5}$$

we obtain the VEVs v, v_Q, a, b, v_ϕ, and χ as

$$\begin{aligned} &v = \frac{m_Q}{\lambda_Q},\\ &v_Q^2 = \tfrac{5}{2}m_Q m_S\lambda_Q^{-2} + \tfrac{5}{4}\lambda_S m_Q^2\lambda_Q^{-3} - g_\phi^2\mu^4\lambda_A\lambda_Q^{-1}(C_1^{-2}-C_2^{-2}),\\ &a = g_\phi\mu^2 C_1^{-1},\\ &b = g_\phi\mu^2 C_2^{-1},\\ &v_\phi^2 = \mu^2,\\ &\chi = -10g_\phi^2\mu^2(\lambda_A m_Q\lambda_Q^{-1}+2m_A)(g_\chi C_1 C_2)^{-1},\end{aligned} \tag{6}$$

where

$$C_1 \equiv 2m_A + 3\lambda_A m_Q\lambda_Q^{-1}, \quad C_2 \equiv 2m_A - 2\lambda_A m_Q\lambda_Q^{-1}. \tag{7}$$

We take the Yukawa couplings $\lambda_Q,\lambda_S,\lambda_A,g_\phi,g_\chi\sim O(1)$, and $m_Q,m_S,m_A,\mu\sim M_G$, which is suggested from the renormalization group analysis on the gauge coupling constants of the low-energy gauge groups [2].

In view of Eq. (3), the colored Higgs H_a ($a=5,\ldots,10$) obtain the GUT scale masses together with Q^{11}_α, but the Higgs H_i ($i=1,\ldots,4$) remain massless as long as $\langle Q^{11}_\alpha\rangle=0$. These massless Higgs multiplets transform as (**2**, **2**) under the $SU(2)_L\times SU(2)_R$ and are identified with two Higgs doublets in the SUSY standard model. The masslessness of H_i is guaranteed by the $U(1)_{A'}$ symmetry, which is an unbroken linear combination of the $U(1)_A$ and a $U(1)$ subgroup of the $SO(10)_{GUT}$.[2] On the other hand, the mass term for the colored Higgs H_a and Q^{11}_α is allowed since they have the $U(1)_{A'}$ charges opposite each other. Note that the presence of the vacuum with unbroken $U(1)_{A'}$ in Eqs. (5) is a dynamical consequence of the present model.

Let us now discuss the gauge coupling constants and gaugino masses at low energies. The SM gauge fields are linear combinations of those of the $SO(10)_{GUT}$ and the $SO(6)_H$, so that one may wonder if the successful gauge coupling unification is spoiled by the presence of hypercolor gauge interactions. However, it is not necessarily true. Assuming that the $SO(10)_{GUT}\times SO(6)_H$ is broken down to the $SU(3)_C\times SU(2)_L\times U(1)_Y$ at the GUT scale for simplicity, the SM gauge couplings are given as

$$\frac{1}{\alpha_3} = \frac{1}{\alpha_{GUT}} + \frac{1}{\alpha_H}, \tag{8}$$

$$\frac{1}{\alpha_2} = \frac{1}{\alpha_{GUT}}, \tag{9}$$

$$\frac{1}{\alpha_1} = \frac{1}{\alpha_{GUT}} + \frac{2}{5}\frac{1}{\alpha_H}, \tag{10}$$

at the GUT scale. Here, α_{GUT} and α_H are the gauge coupling constants of the $SO(10)_{GUT}$ and the $SO(6)_H$. Thus, if hyper-

[2]The $U(1)_{A'}$ symmetry also forbids dangerous dimension-five operators [12] which induce the proton decay through colored Higgs exchanges.

color $SO(6)_H$ is sufficiently strong, $\alpha_H(M_G) \gg \alpha_{GUT}(M_G)$, the GUT unification $\alpha_1(M_G) \simeq \alpha_2(M_G) \simeq \alpha_3(M_G)$ is achieved naturally.[3]

The gaugino masses m_1, m_2, and m_3 are also given by linear combinations of the gaugino masses m_{GUT} and m_H for the $SO(10)_{GUT}$ and the $SO(6)_H$ as

$$\frac{m_3}{\alpha_3} = \frac{m_{GUT}}{\alpha_{GUT}} + \frac{m_H}{\alpha_H}, \tag{11}$$

$$\frac{m_2}{\alpha_2} = \frac{m_{GUT}}{\alpha_{GUT}}, \tag{12}$$

$$\frac{m_1}{\alpha_1} = \frac{m_{GUT}}{\alpha_{GUT}} + \frac{2}{5}\frac{m_H}{\alpha_H}. \tag{13}$$

Here, we have adopted the hidden sector SUSY breaking scenario. This shows that the GUT gaugino mass relation, Eq. (1), can be broken in general. Note that the above equations depend only on the combinations m/α which are invariant under renormalization group at one-loop level, so that these results hold at any scale and are independent of the breaking scales v, v_Q, a, b, and v_ϕ at one-loop level [13].

Next, we discuss phenomenological implications of the above equations [Eqs. (11)–(13)]. We first consider the simplest case where the gaugino masses are originated only from the F term of a dilaton superfield. In this case, the gaugino masses m are universal for all gauge groups ($m_{GUT} = m_H$) at the cutoff scale M_* (string scale or Planck scale). Then, since gauge coupling unification requires that $\alpha_H/\alpha_{GUT}(M_G) \gg 1$, one might think that the deviation from the GUT gaugino mass relation, Eq. (1), is small in view of Eqs. (11)–(13). However, $\alpha_H/\alpha_{GUT}(M_G) \gg 1$ does not mean $\alpha_H/\alpha_{GUT}(M_*) \gg 1$. Indeed, in the present model the hypercolor $SO(6)_H$ is asymptotically free while the $SO(10)_{GUT}$ is not above the GUT scale, so that α_{GUT} and α_H can be comparable at M_*. This implies that both m_{GUT}/α_{GUT} and m_H/α_H can make comparable contributions to the SM gaugino masses even in this simplest case. Moreover, gaugino masses can be nonuniversal in more general cases where gaugino masses arise from F terms of several moduli fields. Thus, we take $m_{GUT}/\alpha_{GUT}(M_G)$ and $m_H/\alpha_H(M_G)$ as independent parameters, hereafter.

In spite of nonunified gaugino masses, Eqs. (11)–(13) suggest that there is one nontrivial relation among gaugino masses for $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$. It is given by eliminating m_{GUT}/α_{GUT} and m_H/α_H as

$$\frac{m_1}{\alpha_1} = \frac{3}{5}\frac{m_2}{\alpha_2} + \frac{2}{5}\frac{m_3}{\alpha_3}. \tag{14}$$

We stress again that this relation holds at any scale and is independent of symmetry breaking scales at the leading order. We have depicted this gaugino mass relation by the solid line in Fig. 1. The horizontal and vertical axes represent

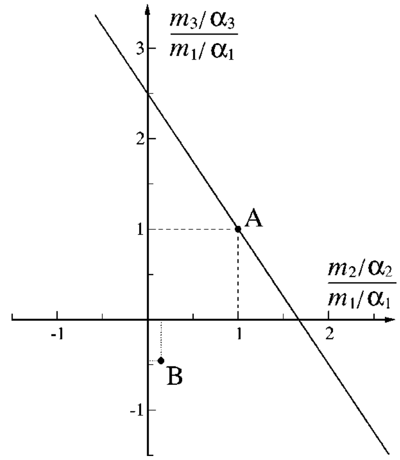

FIG. 1. The gaugino mass relation predicted in the $SO(10)_{GUT} \times SO(6)_H$ model. The point A denotes the case that the GUT gaugino mass relation, $m_1/\alpha_1 = m_2/\alpha_2 = m_3/\alpha_3$, holds. The point B represents the case that the gaugino masses arise only from superconformal anomaly, $m_i/\alpha_i = (b_i/4\pi) m_{3/2}$ ($i = 1,2,3$), where $m_{3/2}$ is the gravitino mass and b_i are the coefficients of one-loop beta functions, $(3, -1, -33/5)$, for $(\alpha_3, \alpha_2, \alpha_1)$.

$(m_2/\alpha_2)/(m_1/\alpha_1)$ and $(m_3/\alpha_3)/(m_1/\alpha_1)$, respectively. The point A denotes the case where the GUT gaugino mass relation, Eq. (1), holds. Here, we have assumed that m_{GUT}/m_H is real in order not to introduce SUSY CP problem. We conclude that the gaugino masses in the present model can deviate from those in the minimal SUSY GUT case but still maintain one relation, Eq. (14). Note, however, that the above gaugino masses are the running gaugino masses at one-loop level, so that the higher-loop effects and threshold corrections which typically induce a few percent contributions [14] should be taken into account when we compare them with the pole masses precisely.[4]

Several comments are in order. First, the gauginos for the SM are linear combinations of those of the $SO(10)_{GUT}$ and the $SO(6)_H$, while the squarks and sleptons purely come from **16** representations of the $SO(10)_{GUT}$. As a result, there may be certain sum rules for squark and slepton masses and they can be used to determine the symmetry breaking pattern and scale [13].[5] Second, although we have assumed that m_{GUT}/m_H is real in our analysis, m_{GUT} and m_H could have small relative phases in general.[6] It may induce observable CP-violating effects and can be used to discriminate the model in future experiments, since this phase cannot be included in the usual SUSY GUT models. In this case, the gaugino masses will slightly deviate from Eq. (14), keeping an inequality $|m_1|/\alpha_1 \leq (3/5)|m_2|/\alpha_2 + (2/5)|m_3|/\alpha_3$. Third, if gaugino masses are generated only by the SM gauge inter-

[3]The correction from the $SO(6)_H$ may explain the slight discrepancy of $\alpha_3(M_Z)$ between the experimental value and the prediction of the minimal SUSY GUT [9].

[4]The gauginos for $SU(2)_L$ and $U(1)_Y$ mix with the Higgsinos resulting in physical particles, charginos, and neutralinos, so that m_1 and m_2 have to be determined experimentally, disentangling these complications [15].

[5]Realistic quark and lepton mass matrices can be obtained by introducing appropriate nonrenormalizable interactions [16,6].

[6]It may be possible that this phase is even of order one [17].

actions at low energies [18] or by superconformal anomalies [19], the gaugino masses generically fall into point A and B in Fig. 1, respectively. In these cases, the present model is not distinguishable from the other GUT models.[7]

[7]There is a gauge mediation model which predicts the same gaugino mass relation as in Eq. (14) due to accidental cancellations of the leading order diagrams responsible for the gaugino masses [20]. This case can be discriminated from the model considered in the text by measuring squark and slepton masses.

To summarize, we have shown that the gaugino masses can deviate from the GUT gaugino mass relation in the SUSY $SO(10)_{GUT}\times SO(6)_H$ model. In spite of nonunified gaugino masses, however, there is one nontrivial relation among the SM gaugino masses, which is independent of symmetry breaking scales at the leading order. Thus, observing the gaugino mass relation (14) in future experiments could test the present model together with the measurement of squark and slepton masses.

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